A 2-3 Symmetry in Neutrino Oscillations

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Abstract

Maximum mixing in atmospheric neutrino oscillation, as well as vanishing of the MNS matrix element U_{e3}, are consequences of a 2-3 symmetry, under which the neutrino mass matrix is invariant under the interchange of second and third generation neutrinos. These predictions of the 2-3 symmetry are consistent with the results of Super-Kamiokande, K2K, and CHOOZ experiments. If the symmetry is exact at a high-energy scale set by right-handed neutrinos, a deviation from these predictions generated by renormalization-group corrections will occur at experimental energies. With an MSSM dynamics, the result can be made to agree with a global fit of the neutrino data, if normal hierarchy is assumed on the neutrino mass spectrum and if the mass of the electron-neutrino is at least about 0.025 eV. The presence of this mass lower bound is a novel and interesting feature of the symmetry that can be falsified by future experiments. Of the three viable solar neutrino solutions, only LMA gives a sizable MNS matrix element U_{e3} that can hopefully be detected in future reactor experiments. Inverted neutrino mass hierarchy is not permitted by this symmetry.

In the basis where the mass matrix of the charged leptons is diagonal, the left-handed flavor-based (symmetric) neutrino mass matrix m' is related to its diagonal form $m = diag[m_1, m_2, m_3]$ by

$$u^T m' u = m, \tag{1}$$

where u is the unitary MNS mixing matrix [1], conventionally parametrized as

$$u = \begin{pmatrix} c_1c_3 & s_1c_3 & s_3 \\ -s_1c_2 - c_1s_2s_3 & c_1c_2 - s_1s_2s_3 & s_2c_3 \\ s_1s_2 - c_1c_2s_3 & -c_1s_2 - s_1c_2s_3 & c_2c_3 \end{pmatrix}. \tag{2}$$

The phases in m_i and u are not measurable in present experiments so we shall assume them to be zero. The parameters in (2) are related to the mixing angles $\theta_1 \equiv \theta_{12}, \theta_2 \equiv \theta_{23}$, and $\theta_3 \equiv \theta_{13}$ by $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. These angles are controlled respectively by the solar, atomospheric, and reactor neutrino oscillations. As in Ref. [2], we take all angles to be between 0 and $\pi/2$. In this article we assume sterile neutrinos to play no role in these oscillations.

Over the years, there have been many attempts to understand the texture of m'. In this paper we assume m' to have an exact symmetry under the interchange of the second and third generation neutrinos, and study its consequences. We shall refer to this symmetry from now on as the 2-3 symmetry. It is a symmetry in which the matrix elements of m' is invariant under the interchange of the flavor basis vectors $|e\rangle \leftrightarrow |e\rangle$ and $|\mu\rangle \leftrightarrow -|\tau\rangle$. The minus sign here is needed to keep the convention of having only positive mixing angles. With this symmetry,

$$\begin{aligned} m'_{e\mu} &= -m'_{e\tau}, \\ m'_{\mu\mu} &= m'_{\tau\tau}, \end{aligned} \tag{3}$$

so we may parametrize m' by four parameters as in

$$m' = \begin{pmatrix} a & b & -b \\ b & f & e \\ -b & e & f \end{pmatrix}. \tag{4}$$

The second constraint in (3) has previously been studied by several authors [3,4].

The mass spectra of quarks and charged leptons suggest that if a generation symmetry is to hold approximately, then it is much more natural to have a 1-2 symmetry [5] rather

than a 2-3 symmetry, because the third-generation fermion is much heavier than those in the first two. For neutrinos, only mass differences are known, but since $(\Delta m_{12})^2 \ll (\Delta m_{23})^2$, a 1-2 symmetry is still more natural if the normal hierarchy $m_1 < m_2 < m_3$ holds with $m_1 \leq |\Delta m_{12}|$. For this reason, the 2-3 symmetry in (3) might seem to be totally unnatural. Nevertheless, it turns out that the 2-3 symmetry on m' given by (3) places no restriction whatsoever on the neutrino masses m_i, so this 2-3 symmetry on m' is not contradictory to a 1-2 symmetry on the mass spectrum. Moreover, the 2-3 symmetry gives rise naturally to the right mixing of neutrinos, as we shall see immediately.

The two conditions in (3) implies $s_3 = 0$ and $s_2 = 1/\sqrt{2}$, and an MNS matrix (2) equal to

$$\begin{aligned} u = \begin{pmatrix} c_1 & s_1 & 0 \\ -\frac{1}{\sqrt{2}}s_1 & \frac{1}{\sqrt{2}}c_1 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_1 & -\frac{1}{\sqrt{2}}c_1 & \frac{1}{\sqrt{2}} \end{pmatrix} = v\cdot w \\ \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{5}$$

To derive this result, note that

$$m'' \equiv v^T m' v = \begin{pmatrix} a & \sqrt{2}b & 0 \\ \sqrt{2}b & f-e & 0 \\ 0 & 0 & f+e \end{pmatrix}, \tag{6}$$

and that m'' can be diagonalized by w. Incidentally, the eigenvalues are $f+e$ and $\frac{1}{2}[a+f-e\pm\sqrt{(a-f+e)^2+8b^2}]$, and the solar mixing angle is determined by the positive solutions of $\cot\theta_1 = [f-e-a\pm\sqrt{(f-e-a)^2+8b^2}]/2\sqrt{2}b$, but these relations are not needed in the following.

The MNS mixing matrix (5) impies maximal mixing in atmospheric neutrino oscillations, consistent with the Super-Kamiokande [6] and the K2K [7] observations. It also gives rise to $u_{e3} = 0$, consistent with the CHOOZ reactor experiment [8]. On the other hand, the solar mixing angle θ_1 and the neutrino masses m_i are unrestricted by (3), as they can be

adjusted by the choice of the four parameters a, b, f, e. Hence it is possible to accommodate the SMA, the LMA, the LOW, or any other solution of solar neutrino oscillations.

If a fundamental symmetry like (3) is present, most likely it occurs at a high-energy scale E_N set by the right-handed neutrinos, and not at present experimental energies. So, strictly speaking, a renormalization-group correction must be applied to (5) before comparing it with experiments. However, if the correction is small, then the 2-3 symmetry is still approximately valid at present energies. This is indeed the case. The corrected result agrees with experimental observations, *for certain scenarios but not for others*. Consequently, if we believe the 2-3 symmetry to be a valid high-energy symmetry, then it can also be used to rule out possible scenarios not directly refuted by current experiments.

The neutrino mass matrix at energy scale E_N that possesses the 2-3 symmetry will continue to be denoted by m'. The renormalization-group corrected neutrino mass matrix at experimental energies will be denoted by M'. It is known that they are related by [9]

$$(M')_{\alpha\beta} = \sqrt{I_\alpha I_\beta}(m')_{\alpha\beta}, \quad (\alpha, \beta = e, \mu, \tau). \tag{7}$$

The quantity I_α is determined from the charged-lepton Yukawa coupling h_α by the relation

$$I_\alpha = \exp[-\int_t^{t_N} dt'\ h_\alpha^2(t')/8\pi^2], \tag{8}$$

where t is the logarithm of the present energy and t_N is the logarithm of E_N. With (7), the symmetry requirement (3) is weakened to read

$$\left(\frac{M'_{e\tau}}{M'_{e\mu}}\right)^2 = \frac{M'_{\tau\tau}}{M'_{\mu\mu}} = \frac{I_\tau}{I_\mu}. \tag{9}$$

Although there are still two relations, the second one is now a restriction on the ratio imposed by the renormalization-group. As a result, the MNS matrix is no longer given by (5), and a deviation from maximal atmospheric mixing and $u_{e3} = 0$ will occur. Nevertheless, since I_α are generally quite close to 1, the deviations, though present, are small and are consistent with a global fit of the data [2]. With a 2-3 symmetry, these small deviations are attributed to renormalization-group effects.

Following Ref. [3], we assume the renormalization-group dynamics to be determined by MSSM with the following parameters: the right-handed neutrino scale $E_N = 10^{13}$ GeV, the GUT unification scale $M_{GUT} = 1.1 \times 10^{16}$ GeV, with a coupling $\alpha_{GUT}^{-1} = 25.64$, and the SUSY scale $M_{SUSY} = 1$ TeV. The Yukawa couplings are taken to be $h_{top} = 3.0$, and $h_{bottom}, h_\tau, h_\mu, h_e$ are determined by the observed fermion masses. With this choice, I_e is always very close to 1 and will be taken as such. I_τ varies from 0.826 at $h_\tau = 3.0$ (corresponding to $\tan\beta = 58.2$) monotonically to 0.99997 at $h_\tau = 0.013$ ($\tan\beta = 1$), while I_μ varies monotonically from 0.9955 to 1.00000 [3]. In this range, the ratio I_τ/I_μ needed in (9) varies from 0.8297 to 0.99997. Ratios beyond this range will be rejected.

The following procedure is employed to test the hypothesis (9), or equivalently, (3). Using upper-case letters to denote (1) at experimental energies, we compute $M' = U^T M U$ from each of the solutions allowed by the global fit in Ref. [2]. The details of this will be discussed in the next paragraph. Then we test whether (9) is obeyed in two steps. First, we vary $\tan^2\theta_3$ between 0 and the approximate upper bound 0.026 set by the CHOOZ reactor experiment [8] to achieve the first equality in (9). If that is not possible the solution is rejected, and a cross ($\times$) is put in Table I. If the solution passes this test, then the value of $\tan^2\theta_3$ is recorded in Table I, and we proceed to compute I_τ/I_μ from the second equality in (9). If the value does not lie between and 0.8297 and 0.99997, we reject the solution and put a cross in Table I. Otherwise the value is recorded in Table I.

The following parameters from atmospheric neutrino experiments will be fixed throughout [2]: $\tan^2\theta_2 = 1.6$, and $\Delta m_{23}^2 = 3 \times 10^{-3}$ eV^2. For solar neutrino observations, we investigate all three viable solutions [2], SMA, LMA, and LOW, with $(\tan^2\theta_1, \Delta m_{12}^2)$ respectively equal to to $(8 \times 10^{-4},\ 5 \times 10^{-6}), (0.4, 3 \times 10^{-5})$, and $(0.8, 10^{-7})$, and Δm_{12}^2 expressed in eV^2. In each of these three cases, the only remaining parameter in U is $tan^2\theta_3$. As for M, we shall consider two scenarios in each case: the normal hierarchy (nh), where $0 \le m_0 \equiv m_1 < m_2 < m_3$, and the inverted hierarchy (ih), where $m_1 > m_2 > m_3 \equiv m_0 \ge 0$. In each of these two scenarios the observed Δm_{12}^2 and Δm_{23}^2 leaves only one free parameter m_0 in the diagonal mass matrix M. So in toto, we have six cases to consider, (SMAnh),

(SMAih), (LMAnh), (LMAih), (LOWnh), (LOWih), each of them having two adjustable parameters, m_0 and $\tan^2\theta_3$. As discussed in the last paragraph, these two parameters are adjusted so that (9) holds. Since I_τ/I_μ can vary over a range, depending on the value of $\tan\beta$, effectively there is one free parameter left, which is taken to be the minimal neutrino mass m_0 in Table I. An upper cutoff of 2 eV is also imposed on the Table, roughly corresponds to the bound observed in the Mainz experiment [10]. One may also want to read the Table with a more stringent upper bound of 0.26 eV in mind, correponding to the experimental limit of neutrinoless double beta decay from the Heidelberg-Moscow experiment [11].

The result is summarized in Table I, for each of the six cases, and a range of m_0 between 0 and 2 eV. We see there that the inverse mass hierarchy (ih) is ruled out by the 2-3 symmetry (3), but with normal mass hierarchy (nh), all three solar neutrino solutions (SMA, LMA, LOW) are allowed. The most striking result of the 2-3 symmetry is that there is a *lower bound* around 0.025 eV for the neutrino masses. This lowerbound increases when $\tan\beta$ decreases from 58.2, which corresponds to $h_\tau = h_{top} = 3.0$. We also see that $\tan^2\theta_3$ are always very small ($\sim 10^{-6}$ or $\sim 10^{-7}$) for the SMA and LOW solutions; only the LMA solution gives a relatively large $\tan^2\theta_3$, of the order 10^{-2} to 10^{-3}.

The 2-3 symmetry has been assumed to hold for the left-handed neutrino mass matrix (4), in the basis where the charged leptons are mass diagonal. By itself the symmetry makes no statement about how right-handed leptons transform, nor even how the left-handed charged leptons behave under a 2-3 transformation. If one assumes that the left-handed charged leptons must transform the same way as the left-handed neutrinos, then in order to keep the charged leptons mass diagonal, these mass terms must break the 2-3 symmetry and the right-handed charged leptons must not transform like the left-handed ones. The dynamical mechanism which could bring this about is presently not understood.

To summarize, neutrino oscillation data with a normal mass hierarchy (nh) are consistent with a 2-3 symmetry (3), at the right-handed neutrino scale of $E_N \sim 10^{13}$ GeV and a renormalization-group extrapolation using MSSM. This symmetry predicts a minimum value

for the electron neutrino mass around 0.025 eV. The actual bound depends on the value of $\tan\beta$; it increases with decreasing $\tan\beta$. The values of $\tan^2\theta_3$ are always tiny for the SMA and the LOW solutions, and they become somewhat substantial ($\sim 10^{-2}$ to 10^{-3}) only in the LMA solution.

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TABLES

Type	m_0 (eV)	$\tan^2\theta_3$	I_τ/I_μ
SMAnh	0.00	0.016	$\times$
	0.02	2.3×10^{-6}	$\times$
	0.02565	1.3×10^{-6}	0.8298
	0.03	9.0×10^{-7}	0.8505
	0.04	5.3×10^{-7}	0.8878
	0.05	3.8×10^{-7}	0.9142
	0.10	2.1×10^{-7}	0.9702
	0.50	1.6×10^{-7}	0.9986
	2.00	1.6×10^{-7}	0.9999
SMAih	0.00	2.1×10^{-13}	$\times$
	2.00	1.6×10^{-7}	$\times$

Type	m_0 (eV)	$\tan^2\theta_3$	I_τ/I_μ
LMAnh	0.00	$\times$	
	0.01	$\times$	
	0.02536	0.0117	0.8298
	0.035	0.0061	0.8715
	0.05	0.0035	0.9143
	0.10	0.0019	0.9702
	0.50	0.0015	0.9986
	1.00	0.0015	0.9997
	2.00	0.0015	0.9999
LMAih	0.00	8.8×10^{-14}	$\times$
	2.00	0.0015	$\times$

Type	m_0 (eV)	$\tan^2\theta_3$	I_τ/I_μ
LOWnh	0.00	$\times$	
	0.001	0.0108	$\times$
	0.02575	1.6×10^{-7}	0.8298
	0.05	4.8×10^{-8}	0.9140
	0.10	2.6×10^{-8}	0.9702
	1.00	2.0×10^{-8}	0.9997
	2.00	2.0×10^{-8}	0.9999
LOWih	0.00	0	$\times$
	2.00	2×10^{-8}	$\times$

TABLE I. Allowed values of the minimal neutrino mass m_0, and the solutions of $\tan^2\theta_3$ and I_τ/I_μ. In the MSSM dynamics considered in this paper, the last column decreases from 0.9997 at $\tan\beta = 1$, to 0.8297 at $\tan\beta = 58.2$, where $m_{top} = m_{tau}$ at M_{GUT}. A cross ($\times$) indicates the absence of a solution at that value of m_0. The first subtable deals with the solar solution SMA, with both normal mass hierarchy (nh) and inverse mass hierarchy (ih). The other two subtables deal with similar cases for the solar solutions LMA and LOW.

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